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Existence of global attractors for a nonlinear wave equation☆

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Abstract

Using a new method developed in [Q. Ma, S. Wang, C. Zhong, Necessary and sufficient conditions for the existence of global attractors for semigroups and applications, Indiana Math. J. 5 (6) (2002) 1542–1558], we prove the existence of the global attractor for a nonlinear wave equation.

Keywords: Absorbing set; Global attractor; *ω-limit compactness*

1. Introduction

There is an extensive literature on the study of wave equations. Many of the studies were concerned with the existence of the absorbing sets and the global attractors (see [2] Ch.Iv, [3–6] and the references therein). But few of the equations involving mixed differential quotient terms. In a recent paper [7], the author considered a initial boundary value problem of a wave equation involving the term u_{tx} as follows:

$$u_{tt} - u_{xx} + \alpha u_{tx} + \beta u_t + f(u) = h(x), \qquad x \in (0, l), t \in \mathbf{R}^+, \tag{1.1}$$

$$u(x, 0) = u_0(x), \qquad u_t(x, 0) = u_1(x), \qquad x \in (0, l), \tag{1.2}$$

$$u(0, t) = u(l, t) = 0, \qquad t \in \mathbf{R}^+, \tag{1.3}$$

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where $u_0 \in \mathbf{V} = H_0^1(0,l)$, u_1 and h are given in $\mathbf{H} = L^2(0,l)$, and α, β are positive constants. Under the conditions that there is a constant $c > 0$ and $f \in C^1(\mathbf{R})$ such that

$$F(s) = \int_0^s f(\eta)\mathrm{d}\eta \geq -c \tag{1.4}$$

and

$$f(s)s - F(s) \geq -c, \tag{1.5}$$

the author proved the existence of an absorbing set in $\mathbf{E_0} = \mathbf{V} \times \mathbf{H}$, but did not give any information about the global attractor. The reason might be that it is difficult to obtain the uniform compactness of the semigroup associated with the problem (1.1)–(1.3). Usually, to do this, one needs to get estimates in a more regular function space by the traditional method (see [2,8,9], for examples). Fortunately, a new method for obtaining the global attractors has been developed in [1]. With this method, one only needs to verify a necessary compactness condition with the same type of energy estimates as those for establishing the absorbing sets.

The aim of this paper is to use the new method to obtain the global attractor for the semigroup associated with the problem (1.1)–(1.3). We prove the result under the conditions (1.4), (1.5) and the following additional hypothesis on f: there is a $0 \leq r < \infty$, such that

$$\lim_{|s|\to\infty} \frac{|f'(s)|}{|s|^r} = 0. \tag{1.6}$$

The paper is organized as follows. At first, in this section, we introduce some results of "the new method" which can be found in [1] and will be used later. Then in Section 2, we prove the existence of the absorbing set and the global attractor in $\mathbf{E_0}$.

Definition 1.1. Let M be a metric space and A be a bounded subset of M. The measure of noncompactness $\gamma(A)$ of A is defined by

$$\gamma(A) = \inf\{\delta > 0 \mid A \text{ admits a finite cover by sets whose diameter } \leq \delta\}.$$

Definition 1.2. A C^0 semigroup $\{S(t)\}_{t\geq 0}$ in a complete metric space M is called *ω-limit compact* if for every bounded subset B of M, and for every $\varepsilon > 0$, there is a $t(B) > 0$, such that

$$\gamma\left(\bigcup_{t\geq t(B)} S(t)B\right) \leq \varepsilon.$$

Condition (C). For any bounded set B of a Banach space X and for any $\varepsilon > 0$, there exists a $t(B) > 0$ and a finite dimensional subspace X_1 of X such that $\{\|P_m S(t)B\|\}$ is bounded and

$$\|(I - P_m)S(t)x\| < \varepsilon, \qquad \text{for } t \geq t(B), x \in B,$$

where I is the identity and $P_m : X \to X_1$ is a bounded projector.

Theorem 1.3. *Let $\{S(t)\}_{t\geq 0}$ be a C^0 semigroup in a complete metric space M. Then $\{S(t)\}_{t\geq 0}$ has a global attractor $\mathcal{A}$ in M if and only if*

(1) *$\{S(t)\}_{t\geq 0}$ is ω-limit compact, and*
(2) *there is a bounded absorbing set $B \subset M$.*

Lemma 1.4. *Let X be a Banach space and $\{S(t)\}_{t\geq 0}$ be a C^0 semigroup in X.*

(1) *If Condition (C) holds, then $\{S(t)\}_{t\geq 0}$ is ω-limit compact.*
(2) *Let X be a uniformly convex Banach space. Then $\{S(t)\}_{t\geq 0}$ is ω-limit compact if and only if Condition (C) holds.*

Theorem 1.5. *Let X be a uniformly convex Banach space (especially a Hilbert space). Then the C^0 semigroup $\{S(t)\}_{t\geq 0}$ has a global attractor if and only if*

(1) *there is a bounded absorbing set $B \subset M$.*
(2) *Condition (C) holds.*

2. Absorbing set and global attractor in $\mathbf{E_0}$

Let the denotations of $\mathbf{V}$, $\mathbf{H}$ and $\mathbf{E_0}$ be the same as in the introduction, and $(\cdot,\cdot)$, $\|\cdot\|$ denote the inner product and norm in $\mathbf{H}$ respectively, i.e., for any $u, v \in \mathbf{H}$,

$$(u, v) = \int_0^l uv\mathrm{d}x, \qquad \|u\| = \left(\int_0^l |u|^2\mathrm{d}x\right)^{\frac{1}{2}}.$$

The norm in $\mathbf{V}$ is $\|u\|_{\mathbf{V}} = \|u_x\|$ and we have

$$\|u_x\|^2 \geq \lambda_1\|u\|^2,$$

where λ_1 is the first eigenvalue of $-u_{xx}$ with Dirichlet boundary condition.

Lemma 2.1 *([7] Theorem 1). Let $u_0, u_1, h, \alpha, \beta, c$ be given. Suppose $f \in C^1(\mathbf{R})$ satisfies the conditions (1.4) and (1.5). Suppose further that the problem (1.1)–(1.3) has a unique weak solution and $S(t), t > 0$, defined by $S(t)(u_0, u_1) = (u(t), u_t(t))$, is the semigroup generated by the problem (1.1)–(1.3). Then $S(t), t > 0$ has a bounded absorbing ball.*

Remark 2.2. In [7], the continuous semigroup $S(t)$ was said to be defined for $t > 0$ and to satisfy $S(t + s) = S(t)S(s)$. In fact, the continuous semigroup $S(t)$ can be defined for $t \geq 0$, satisfies $S(t + s) = S(t)S(s)$ and $S(0) = Id$. So we will denote the semigroup as $\{S(t)\}_{t\geq 0}$ in the following, and $\mathbf{R}^+$ should be understand as $[0, \infty)$.

Remark 2.3. To prove Lemma 2.1 is equivalent to prove the existence of an absorbing set in $\mathbf{E}_0$. Although the lemma has been proved in [7], we still give a proof here. Our proof is different from that in [7] but is similar to that in [2]. We adopt and present the proof also because we will use the same method to verify Condition (C) and obtain the existence of the global attractor.

Proof of Lemma 2.1. Take the inner product of (1.1) in $\mathbf{H}$ with $v = u_t + \delta u$, $0 < \delta \leq \delta_0$; δ_0 will be chosen later. After computation and using the conditions (1.4) and (1.5), we obtain

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\left(\|v\|^2 + \|u_x\|^2 + 2\int_0^l F(u)\mathrm{d}x\right) + (\beta - \delta)\|v\|^2 + \delta\|u_x\|^2$$
$$- \delta(\beta - \delta)(u, v) - \alpha\delta(u_x, v) + \delta\int_0^l f(u)u\mathrm{d}x = (h, v). \tag{2.1}$$

Now, using the Hölder inequality, Young inequality and the condition (1.5), we have the estimate

$$\begin{aligned}
&(\beta-\delta)\|v\|^2+\delta\|u_x\|^2-\delta(\beta-\delta)(u,v)-\alpha\delta(u_x,v)+\delta\int_0^l f(u)u\mathrm{d}x\\
&\quad\ge(\beta-\delta)\|v\|^2+\delta\|u_x\|^2-\beta\delta\|u\|\|v\|-\alpha\delta\|u_x\|\|v\|+\delta\int_0^l(F(u)-c)\mathrm{d}x\\
&\quad\ge(\beta-\delta)\|v\|^2+\delta\|u_x\|^2-\frac{\beta^2\delta}{\lambda_1}\|v\|^2-\frac{\delta}{4}\|u_x\|^2-\alpha^2\delta\|v\|^2-\frac{\delta}{4}\|u_x\|^2+\delta\int_0^l(F(u)-c)\mathrm{d}x.
\end{aligned}$$

Choose δ_0 such that

$$\delta_0\left(1+\frac{\beta^2}{\lambda_1}+\alpha^2\right)=\frac{\beta}{2}. \tag{2.2}$$

Since $1+\frac{\beta^2}{\lambda_1}+\alpha^2>1$, and $0<\delta\le\delta_0$, we have

$$\beta-\delta\left(1+\frac{\beta^2}{\lambda_1}+\alpha^2\right)\ge\frac{\beta}{2}\ge\delta. \tag{2.3}$$

It follows that

$$\begin{aligned}
&\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\left(\|v\|^2+\|u_x\|^2+2\int_0^l(F(u)+c)\mathrm{d}x\right)+\frac{\beta}{2}\|v\|^2+\frac{\delta}{2}\|u_x\|^2+\delta\int_0^l(F(u)+c)\mathrm{d}x\\
&\quad\le 2\delta cl+\|h\|\|v\|\le 2\delta cl+\frac{\|h\|^2}{\beta}+\frac{\beta}{4}\|v\|^2,
\end{aligned}$$

and then

$$\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}t}\left(\|v\|^2+\|u_x\|^2+2\int_0^l(F(u)+c)\mathrm{d}x\right)+\delta\left(\|v\|^2+\|u_x\|^2+2\int_0^l(F(u)+c)\mathrm{d}x\right)\\
&\quad\le 4\delta cl+\frac{2\|h\|^2}{\beta}.
\end{aligned} \tag{2.4}$$

Set

$$y(t)=\|v(t)\|^2+\|u_x(t)\|^2+2\int_0^l(F(u(t))+c)\mathrm{d}x.$$

From condition (1.4) we know $y(t)\ge 0$ and (2.4) can be rewritten as

$$\frac{\mathrm{d}}{\mathrm{d}t}y(t)+\delta y(t)\le 4\delta cl+\frac{2\|h\|^2}{\beta}. \tag{2.5}$$

By Gronwall's inequality, we obtain

$$y(t)\le y(0)\mathrm{e}^{-\delta t}+\left(4cl+\frac{2\|h\|^2}{\beta\delta}\right)(1-\mathrm{e}^{-\delta t}),\qquad t\ge 0. \tag{2.6}$$

For any bounded subset B of $\mathbf{E_0}$, $(u_0,u_1)\in B$, $\{\int_0^l F(u_0)\mathrm{d}x\}$ is bounded, too. Hence

$$R=R(B)=\sup_{(u_0,u_1)\in B}y(0)=\sup_{(u_0,u_1)\in B}\left\{\|u_0\|_{\mathbf{V}}^2+\|u_1+\delta u_0\|_{\mathbf{H}}^2+2\int_0^l(F(u_0)+c)\mathrm{d}x\right\}<\infty$$

and

$$\lim_{t\to\infty}\sup_{(u_0,u_1)\in B} y(t) \le 4cl + \frac{2\|h\|^2}{\beta\delta} \equiv \mu_0^2. \tag{2.7}$$

Let $\mu_1 > \mu_0$ be fixed, and

$$t_0 = t_0(R,\mu_1) = \frac{1}{\delta}\ln\frac{R}{\mu_1^2-\mu_0^2} \tag{2.8}$$

for any $t \ge t_0$, we have $y(t) \le \mu_1^2$ and

$$\begin{aligned}\|u(t)\|_{\mathbf{V}}^2 + \|u_t(t)\|_{\mathbf{H}}^2 &= \|u(t)\|_{\mathbf{V}}^2 + \|u_t(t)+\delta u(t)-\delta u(t)\|_{\mathbf{H}}^2\\
&\le \|u(t)\|_{\mathbf{V}}^2 + 2(\|u_t(t)+\delta u(t)\|_{\mathbf{H}}^2 + \delta^2\|u(t)\|_{\mathbf{H}}^2)\\
&\le \|u(t)\|_{\mathbf{V}}^2 + 2\left(\|u_t(t)+\delta u(t)\|_{\mathbf{H}}^2 + \frac{2\delta^2}{\lambda_1}\|u(t)\|_{\mathbf{V}}^2\right)\\
&\le 2\left(1+\frac{\delta^2}{\lambda_1}\right)(\|u(t)\|_{\mathbf{V}}^2 + \|u_t(t)+\delta u(t)\|_{\mathbf{H}}^2)\\
&\le 2\left(1+\frac{\delta^2}{\lambda_1}\right)y(t)\\
&\le 2\left(1+\frac{\delta^2}{\lambda_1}\right)\mu_1^2,\end{aligned}$$

so we obtain

$$\|u(t)\|_{\mathbf{V}}^2 + \|u_t(t)\|_{\mathbf{H}}^2 \le \rho_0^2, \qquad \text{for } t \ge t_0, \tag{2.9}$$

where

$$\rho_0^2 = 2\left(1+\frac{\delta^2}{\lambda_1}\right)\mu_1^2.$$

This indicates that the ball of $\mathbf{E_0}$, $B_0 = B_{\mathbf{E_0}}(0,\rho_0)$, centered at 0 of radius ρ_0, is an absorbing set. The proof is completed. □

Lemma 2.4. *Assume the nonlinear term $f : \mathbf{R} \mapsto \mathbf{R}$ is continuous and satisfies* (1.6). *Then the nonlinear operator $f : \mathbf{V} \mapsto \mathbf{H}$ is continuous and compact.*

Proof. Let $\{u_n\}$ be a bounded sequence in $\mathbf{V} = H_0^1(0,l)$. By the Soblev embedding theorem, $\{u_n\}$ is bounded in $L^k(0,l)$ for any $1 \le k < \infty$, and has a convergent subsequence in $L^2(0,l)$. Without loss of generality, we assume $\{u_n\}$ converges to u^* in $\mathbf{H} = L^2(0,l)$. It is sufficient to prove that $\{f(u_n)\}$ converges to $\{f(u^*)\}$ in $\mathbf{H}$.

For any $\varepsilon > 0$, we infer from (1.6) that

$$|f'(s)|^2 \le \varepsilon|s|^{2r} + C_\varepsilon. \tag{2.10}$$

So we have, for some $0 < \theta < 1$,

$$\begin{aligned}\int_0^l |f(u_n) - f(u^*)|^2\mathrm{d}x &= \int_0^l |f'(u^*+\theta(u_n-u^*))|^2|u_n-u^*|^2\mathrm{d}x\\
&\le \varepsilon\int_0^l |u^*+\theta(u_n-u^*)|^{2r}|u_n-u^*|^2\mathrm{d}x + C_\varepsilon\int_0^l |u_n-u^*|^2\mathrm{d}x\end{aligned}$$

$$
\begin{aligned}
&= \varepsilon \int_0^l |u^*(1-\theta) + \theta u_n|^{2r} |u_n - u^*|^2 \mathrm{d}x + C_\varepsilon \int_0^l |u_n - u^*|^2 \mathrm{d}x \\
&\leq 4^r \varepsilon \int_0^l (|u^*|^{2r} + |u_n|^{2r}) |u_n - u^*|^2 \mathrm{d}x + C_\varepsilon \int_0^l |u_n - u^*|^2 \mathrm{d}x \\
&\leq 4^r \varepsilon \left[\left(\int_0^l |u^*|^{2rp} \mathrm{d}x \right)^{\frac{1}{p}} + \left(\int_0^l |u_n|^{2rp} \mathrm{d}x \right)^{\frac{1}{p}} \right] \left(\int_0^l |u_n - u^*|^{2q} \mathrm{d}x \right)^{\frac{1}{q}} \\
&\quad + C_\varepsilon \int_0^l |u_n - u^*|^2 \mathrm{d}x,
\end{aligned}
$$

where $p > 0$, $q > 0$ and $\frac{1}{p} + \frac{1}{q} = 1$.

Since $\{u_n\}$ is bounded in $L^k(0,l)$ for any $1 \leq k < \infty$, taking $n \to \infty$ and letting $\varepsilon \to 0$. It follows from the above inequalities that

$$
\lim_{n \to \infty} \int_0^l |f(u_n) - f(u^*)|^2 \mathrm{d}x = 0.
$$

This complete the proof. □

Theorem 2.5. *Assume $f : \mathbf{R} \mapsto \mathbf{R}$ is continuous and satisfies* (1.4)–(1.6). *Then the semigroup $\{S(t)\}_{t \geq 0}$ associated with problem* (1.1)–(1.3) *possesses a global attractor $\mathcal{A}$ in $\mathbf{E_0}$.*

Proof. By Theorem 1.5 and Lemma 2.1, we only need to verify Condition (C).

Let λ_j be the eigenvalues of $-u_{xx}$ and w_j be the corresponding eigenvectors, $j = 1, 2, \cdots$. Recall that $\lambda_1 < \lambda_2 \leq \cdots$, and $\lambda_m \to \infty$ as $m \to \infty$.

It is well know that $\{w_j\}_{j=1}^{\infty}$ form an orthogonal basis of $\mathbf{H}$. We write

$$
\mathbf{H}_m = \mathrm{span}\{w_1, \ldots, w_m\}.
$$

Since $h \in \mathbf{H}$ and $f : \mathbf{V} \mapsto \mathbf{H}$ is compact (which has been verified in Lemma 2.4), for any $\varepsilon > 0$, there exists some $m \in \mathbf{N}$ such that

$$
\|(I - P_m)h\| \leq \frac{\varepsilon}{2}, \tag{2.11}
$$

$$
\|(I - P_m) f(u)\| \leq \frac{\varepsilon}{2}, \qquad \text{for } u \in B_{\mathbf{V}}(0, \rho_0), \tag{2.12}
$$

where $P_m : \mathbf{H} \mapsto \mathbf{H_m}$ is an orthogonal projection and ρ_0 is the radius of the absorbing set obtained in Lemma 2.1. For any $(u, u_t) \in \mathbf{E_0}$, we write

$$
(u, u_t) = (P_m u, P_m u_t) + ((I - P_m)u, (I - P_m)u_t) = (u_1, u_{1t}) + (u_2, u_{2t}). \tag{2.13}
$$

Take the inner product of (1.1) in $\mathbf{H}$ with $v_2 = u_{2t} + \delta u_2$, $0 < \delta \leq \delta_1$. After a computation like in the proof of Lemma 2.1, we obtain

$$
\begin{aligned}
&\frac{1}{2} \frac{\mathrm{d}}{\mathrm{d}t} (\|v_2\|^2 + \|u_{2x}\|^2) + (\beta - \delta)\|v_2\|^2 + \delta \|u_{2x}\|^2 \\
&\quad - \delta(\beta - \delta)(u_2, v_2) - \alpha\delta(u_{2x}, v_2) = -(f(u), v_2) + (h, v_2).
\end{aligned} \tag{2.14}
$$

This is the same as in the proof of Lemma 2.1, except for a replacement of λ_1 with λ_{m+1} and, correspondingly, a choice of δ_1, such that

$$\delta_1 \left(1 + \frac{\beta^2}{\lambda_{m+1}} + \alpha^2\right) = \frac{\beta}{2};$$

then we have

$$(\beta - \delta)\|v_2\|^2 + \delta\|u_{2x}\|^2 - \delta(\beta - \delta)(u_2, v_2) - \alpha\delta(u_{2x}, v_2) \geq \frac{\beta}{2}\|v_2\|^2 + \frac{\delta}{2}\|u_{2x}\|^2. \tag{2.15}$$

Combining with (2.12) and (2.13), it follows from (2.14) and (2.15) that

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}(\|v_2\|^2 + \|u_{2x}\|^2) + \frac{\beta}{2}\|v_2\|^2 + \frac{\delta}{2}\|u_{2x}\|^2 \leq \varepsilon\|v_2\| \leq \frac{\beta}{4}\|v_2\|^2 + \frac{\varepsilon^2}{\beta},$$

which implies

$$\frac{\mathrm{d}}{\mathrm{d}t}(\|v_2\|^2 + \|u_{2x}\|^2) + \delta(\|v_2\|^2 + \|u_{2x}\|^2) \leq \frac{2\varepsilon^2}{\beta},$$

by the Gronwall's inequality, we obtain, for $t \geq t_0$,

$$\begin{aligned}\|v_2(t)\|^2 + \|u_{2x}(t)\|^2 &\leq (\|v_2(t_0)\|^2 + \|u_{2x}(t_0)\|^2)\mathrm{e}^{-\delta(t-t_0)} + \frac{2\varepsilon^2}{\beta\delta}(1 - \mathrm{e}^{-\delta(t-t_0)}) \\ &\leq \rho_0\mathrm{e}^{-\delta(t-t_0)} + \frac{2\varepsilon^2}{\beta\delta},\end{aligned}$$

where t_0 is given in (2.8). Taking

$$t_1 - t_0 = \frac{1}{\delta}\ln\frac{\rho_0^2}{\varepsilon^2},$$

then

$$\|v_2(t)\|^2 + \|u_{2x}(t)\|^2 \leq \varepsilon^2\left(1 + \frac{2}{\beta\delta}\right), \qquad \text{for } t \geq t_1$$

and

$$\|u_2(t)\|_{\mathbf{V}}^2 + \|u_{2t}(t)\|_{\mathbf{H}}^2 \leq 2\left(1 + \frac{\delta^2}{\lambda_{m+1}}\right)(\|u_2(t)\|_{\mathbf{V}}^2 + \|v_2(t)\|_{\mathbf{H}}^2) \leq C^2\varepsilon^2,$$

where

$$C^2 = 2\left(1 + \frac{2}{\beta\delta}\right)\left(1 + \frac{\delta^2}{\lambda_{m+1}}\right).$$

This show that Condition (C) is satisfied, and the proof is completed. □

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